**Motivation**

* truth tables have limitations
  + number of rows grows exponentially in number of atomic sentences
  + can only detect tautological consequence, not logical consequence (not all of them at least), because only takes into account truth-functional connectives
  + many types of reasoning rely on logic of other kinds of expressions
* **methods of proof** both formal and informal give us the required extensibility
* we will develop the notion that proof methods are an alternative to truth tables with the advantage that they can be used even when the validity of the proof depends on more than just the Boolean operators
* Boolean connectives give rise to many patterns of inference

**Valid Inference Steps**

* **rule of thumb:** in an **informal proof** it is always legitimate to move from a sentence P to another sentence Q if both you and your “audience” already know that Q is a logical consequence of P

**Three Simple Inference Steps** that use the principle that you can use already established cases of logical consequence in informal proofs

***conjunction elimination (simplification)***

* we are justified in inferring from a conjunction of any number of sentences, any one of its conjuncts

***conjunction introduction***

* if we want to prove a conjunction of a bunch of sentences, we may do so by proving each conjunct separately

***disjunction introduction***

* if we have proven some sentence P, then we can infer any disjunction that has P as one of its disjuncts: if P is true then so is any such disjunction

**Matters of Style**

* every step in a “good” proof, besides being correct, should have two properties
  + it should be **easily understood** and **significant**
* **easily understood:** other people should be able to follow the step without undue difficulty: they should be able to see that the step is valid without having to engage in a piece of complex reasoning on their own
* **significant:** the step should be informative, not a waste of the reader’s time
* typically, the more significant the step, the harder it is to follow; good style requires a reasonable balance between the two

Boolean connectives give rise to two entirely new methods of proof that can be applied in all types of rigorous reasoning. The first method is proof by cases.

**Proof by Cases,** aka **disjunction elimination**

* far more significant than the inference steps introduced above
* begin with a desired goal we want to prove, S, and a disjunction we know to be true, P1 | P2 | … | Pn
* consider n cases
  + in each case, we assume one of the disjuncts to be true, from P1 to Pn
* if we are able to prove our desired result S in each of these cases, we are justified in concluding that S holds
* the validity of proof by cases cannot be demonstrated by the simple truth table method we saw in Chapter 4
* we infer the conclusion S from the fact that S is provable from each of the disjuncts P and Q
  + this relies on the principle that if S is a logical consequence of P and also a logical consequence of Q, then it is a logical consequence of P | Q

**Indirect Proof: Proof by Contradiction**

* one of the most important methods of proof
* aka **indirect proof,** or **reductio ad absurdum;** these are the names for the informal technique
* the formal name, its counterpart in FOL is called **negation introduction**
* **basic idea**
  + we want to prove a negative sentence, say ~S, from some premises P1, …, Pn
  + temporarily assume S and show that a contradiction follows from this assumption: ie that S is always false when the premises are true. If we do this it means that ~S is always true when the premises are true. Therefore, S is a logical consequence of the premises.

**Contradiction**

* intuitively, any claim that cannot possibly be true, or any set of claims which cannot all be true simultaneously
* **contradictory** or **inconsistent** set of sentences: any set of sentences that could not all be true in any single situation
* ⊥ is symbol used as short-hand way of saying that a contradiction has been obtained
  + it means that a conclusion has been reached which is logically impossible, or that several conclusions have been derived which, taken together, are impossible
* a sentence S is a logical impossibility if and only iff its negation ~S is logically necessary

**Inconsistent Set of Premises**

* there is no situation that makes all of the premises true
* any sentence S is a logical consequence of an inconsistent set of premises
  + there is no situation that makes the premises true and S false, simply because there’s no situation making the premises true
  + therefore S is true in every situation that makes the premises true, namely in no situation at all
* An argument from inconsistent premises is always valid, but can never be sound, since the premises can never all be true
* An unsound argument gives exactly the same support for its conclusion as an invalid argument: exactly none.